TIME VALUE OF MONEY

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Introduction – Time Value of Money

The concept of time value of money can be best explained by the fact that a dollar in hand today is worth more than a dollar to be received at some time in the future. This is because if you have a dollar in hand you can invest it to earn income. You lose this income over the period of time between now and the date you may receive a dollar in the future.

In investing and borrowing the difference between the value of the dollar now (PRESENT VALUE) and the value of the dollar at a future date (FUTURE VALUE) is INTEREST.

The amount of interest charged depends on many factors, some of which are:

- **The certainty or lack of certainty of receiving the interest and, in the case of loans, the return of the amount loaned (safety).**
- **The cost of collecting and servicing a loan.**
- **The amount of interest that could be earned in other types of transactions (competition).**
- **For lenders, the cost of money. For example, the higher the rate a bank has to pay to attract savings deposits, the more they have to charge on loans.**
- **Your estimate of inflation during the period.**

This concept is developed further in other sections.

Overview

All financial calculations are composed of four basic elements:

- **Present Value**
- **Future Value**
- **Interest (or Interest Rate)**
- **Time**

In addition, in many instalment transactions, the Payment Schedule must be taken into consideration. Five common types of payment schedules are:

- **Non-Annuity**
- **Equal Payment**
- **Unequal and/or Irregular Payments**
- **Balloon Notes**
- **Equal Principal Payments**

Present and Future Value

In investing and borrowing the difference between the value of the dollar now (PRESENT VALUE) and the value of the dollar at a future date (FUTURE VALUE) is INTEREST.

To relate these to common transactions:

- **You deposit $100 in a savings account. This is the Present Value. During the year, the account earns $5.12 Interest. At the end of the year the account balance is $105.12. This is the Future Value.**
- **You purchase an automobile on time payments, or purchase a home and take out a mortgage. The amount financed (purchase price less down payment) is the Present Value.**
- **You buy a $10,000 Treasury bill with a six month maturity. The treasury sells you the bill for the discounted amount of $9,852 (Present Value). At the end of the six months the treasury pays you the face amount of $10,000 (Future Value). The difference of $148 is the interest you earned.**

Interest or Interest Rate

The interest rate to be applied to the time value calculation is, of course, the factor that determines the difference between the Present Value and the Future Value, or the earnings on a savings or loan transaction.

Rates are usually expressed as an annual percent – for example .06 = 6%.

Therefore, you may imagine that the calculation of interest would be straightforward: $100 invested for 1 year at 6% yields $6. However, in practice, the yield can vary greatly depending on compounding periods and, in the case of instalment transactions the payment schedule. COMPOUNDING, COMPOUNDING FREQUENCY and PAYMENT SCHEDULES are discussed in following sections

APR and APY.

In an attempt to enable consumers to better compare different lending and savings offers, the government has legislated that lending institutions must express their rate as an Annual Percentage Rate (APR), and savings institutions must express their rate as Annual Percentage Yield (APY). There are complicated rules for computing APR and APY which need not concern us. The important thing is that all institutions must compute them the **same way** so that comparisons will be valid.

Time

The amount of time elapsed between the beginning of a transaction and the end of a transaction is essential in time-value calculations.

We are all familiar with the basic equation used to determine interest:

> Interest is equal to Principal times Rate times Time, or I = PRT.
> If you lend $1000 for one year at a rate of 6%, the equation is I = 1000 x .06 x 1, or $60.

In the above transaction there was only one compounding period – the end of the year. However, in most transactions there will be several compounding periods during the period of the transaction – in those cases Time is usually expressed as the number of COMPOUNDING PERIODS.

Compounding

As stated in the Rate section, the simple annual rate expressed as a percentage usually does not reflect the actual rate or interest amount in a transaction. You must also know the COMPOUNDING FREQUENCY.

The concept of compounding of interest can best be understood by an example of a savings account. Suppose you deposit $1,000 into a savings account (the Present Value). The bank should tell you, in addition to the rate of interest paid, how often the interest is compounded. When a compounding period occurs, the interest is credited to the account and from that time on you earn interest on the sum of the deposit plus the interest credited.

Assume that the interest is compounded monthly and the annual rate is 12% (1% per month, or 1% per compounding period). At the end of the first month, the bank adds 1% of $1,000, or $10 to the account. Now you are earning interest on $1,010. The next month the bank adds 1% of $1,010, or $10.10 to the account. Now you are earning interest on $1,020.10 – and so on. At the end of the year the balance will be $1,126.83 (the Future Value). You will have earned $126.83 in interest, which is clearly more than 12% of $1,000, which would be $120. Dividing $126.83 by $1000 gives .12683, or 12.683%. This is known as the EFFECTIVE RATE of the transaction.

Compounding Frequency – Continuous Compounding

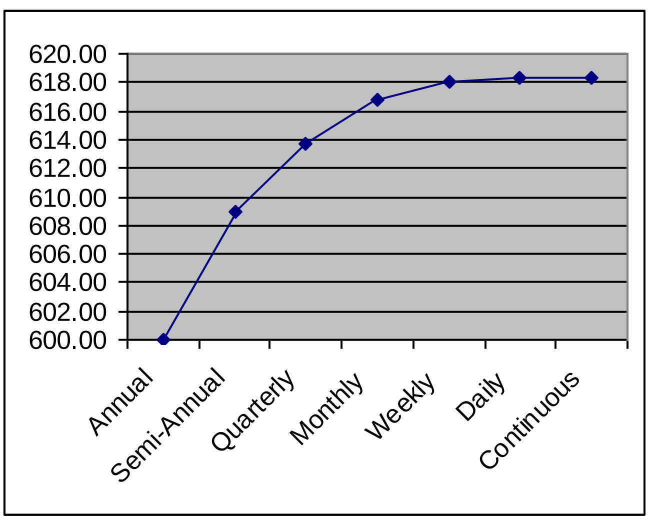
From the previous section on compounding, it is clear that the more often a loan or savings account is compounded, the more the lender or you will earn. At times of competition for savings dollars some institutions have offered the ultimate – **CONTINUOUS COMPOUNDING.** This means that rather than being compounded daily, or hourly, the number of compounding periods is increased to infinity.

> As this is written (March 2004) interest rates are at a 50 year low with the discount rate[1] at 1%. It may seem as though competition for savings dollars is a far-fetched idea. However, in the 79 days from 10/1/1980 till 12/19/1980 the prime rate[2] increased 12 times reaching a high of 21.50%. At that time there was an intense competition to see who could offer the highest rates on savings! No one (that I have found) can predict the future of interest rates.

This isn't as good as it sounds, however. The more often a deposit is compounded, the less additional money you will earn. For example, assume $10,000 deposited for a year at 6%. The total interest earned with various compounding periods is:

Compounding Period	Total Interest Earned	Increment
Annual	$ 600.00	
Semi-Annual	$ 609.00	$ 9.00
Quarterly	$ 613.64	$ 4.64
Monthly	$ 616.78	$ 3.14
Weekly	$ 618.00	$ 1.22
Daily	$ 618.31	$ .31
Continuous	$ 618.37	$ .06

Expressed graphically, you can see that increasing compounding periods rapidly increases earnings in the beginning, but the increases flatten as the periods increase.



[1] The rate at which member banks may borrow short term funds directly from a Federal Reserve Bank.

[2] The interest rate banks charge their most creditworthy commercial customers.

Payment Schedule and Compounding

The relationship of payment frequency and compounding frequency must also be considered. This applies both to borrowing/lending transactions and savings transactions.

BORROWING/LENDING TRANSACTIONS.

Assume you wish to borrow $10,000 for one year at a rate of 6% with payments at the end of the payment periods. The lending institution usually offers terms of monthly payments with monthly compounding. However, you wish to pay in two semi-annual instalments rather than monthly. If the lending institution agrees to this payment schedule, the question still remains – how shall the interest be compounded? If the lender agrees to compounding at the same time as the payments (two compounding periods per year) the transaction would be as follows:

2 semi-annual instalments of $ 5,226.11

Total interest: $ 452.22

However, the lending institution would probably not want to lose the yield gained from monthly compounding (twelve compounding periods per year). Therefore, it would offer semi-annual payments, but with monthly compounding. This transaction would be:

2 semi-annual instalments of $ 5,228.97

Total interest: $ 457.94

The increase in interest earned to the lender would be $ 5.72 – not much for the individual transaction, but significant if multiplied by many transactions.

SAVINGS TRANSACTIONS.

The same considerations would apply if you wished to make periodic deposits in a savings account. Assume quarterly deposits of $500 at an interest rate of 6%, with deposits at the beginning of periods. If the institution compounds interest quarterly you would earn $76.13 at the end of one year – your total balance would be $2,076.13. However, with monthly compounding, your earned interest at the end of one year would be $76.52, an increase of 39 cents. Again, not much cash but pennies count!

Types of Payment Transactions

Non-Annuity

Since ANNUITIES imply a series of payments, non-annuity calculations reflect situations where there are no periodic payments. Instead, an initial sum of money (PRESENT VALUE) is affected by collection or payment of interest during a given period, resulting in a final sum (FUTURE VALUE).

Common transactions involve savings deposits or purchase of Treasury bills.

Equal Payment

Transactions with equal payments and equal payment periods are the most common type of time value calculations encountered by the average person.

This type of transaction includes time payment contracts, such as for autos and merchandise, and mortgage transactions. Some types of leasing transactions can fit this format although usually they are better served by the unequal payment or balloon formats.

In equal payment transactions the compounding period usually is the same as the payment period, especially for monthly payments. Payments at other intervals, such as quarterly, semi-annually, annually may, however, have compounding periods more frequent than payment periods. There is also a possibility of continuous compounding (previously discussed).

Interest calculations take into consideration whether the payment is at the beginning or end of the payment period. Most equal payment instalment transactions have payments at the end of each payment period; however they may be at the beginning in such transactions as savings or leases.

Unequal Payments and/or Frequencies – Internal Rate of Return

There are types of transactions where the payments are not in equal amounts, and may not occur at a regular frequency.

In this type of transaction the borrower or lender usually wishes to determine the rate or, as it is more commonly called, the INTERNAL RATE OF RETURN or IRR, where the Present Value or Future Value is known.

In all cases, the payment schedule must be known. It is frequently helpful to construct a cash flow diagram to represent cash inflows or outflows. The transaction should be viewed from the point of view of the lender, and the Present Value is assumed to be a negative cash flow. Also, a payment frequency must be established, even though there may be frequency periods in which no cash flow occurs – in such cases a payment of zero is shown.

For example, assume a loan of $4,000, which is to be repaid in annual, end of period, payments, with annual compounding. No money is paid at the end of the first year. The following amounts are paid at the end of succeeding years: $200, $500, $1,000, $4,000.

The cash flow diagram would be:

Year	**Cash Flow**
0	($ 4,000) – Present Value
1	$ 0.00
2	$ 200,00
3	$ 500.00
4	$ 1,000.00
5	$ 4,000.00

Using the above schedule the Internal Rate of Return would be 8.154%.

Balloon Payments

Balloon payment calculations assume a series of equal payments with one payment in a different amount (usually larger), known as the BALLOON.

In most cases, the balloon is due at the same time as the final regular equal payment, or at the end of the period following the final regular equal payment.

Note that if the balloon is due at the same time as a regular payment, the total amount due is the **sum** of the two.

An automobile lease transaction implies that a balloon is due at the end of the transaction when you may purchase the vehicle for the calculated residual value.

Equal Principal Payments

When an equal or constant amount is paid to principal of a loan each period, plus the amount of accumulated interest for the period, the total amount of each payment is different from the other payments.

The interest component of each payment declines over the term of the loan; therefore the periodic payment amount also declines over the term.

Beware of Prepayment Penalties

Prepayment penalties may be inserted in a contract by the lender. They provide that if the loan is paid off in advance, a penalty may be charged.

A prepayment penalty can substantially increase the disclosed Annual Percentage Rate (APR). The disclosed APR is calculated based on the assumption that the complete payment schedule will be followed.

For that reason you should consider carefully if you might prepay the loan within the period within which a prepayment penalty will be charged.

Prepayment penalties may be calculated in many ways. A typical provision might be that if a mortgage is prepaid within the first five years a penalty of 4% of the amount financed will be charged.

Assume a 20 year mortgage of $200,000 at a disclosed APR of 5%. The monthly payment on such a mortgage would be $1,319.91. The contract provides that if the mortgage is prepaid within the first five years a penalty of 4% of the amount financed ($8,000) will be charged.

The following table will show the actual APR if the mortgage is prepaid at the end of the first through fourth years.

Month Prepaid	**Actual Annual Percentage Rate**
12th month	8.89%
24th month	6.92%
36th month	6.27%
48th month	5.94%

From the above it can be seen that the earlier a contract with a prepayment penalty is prepaid, the larger the rate of return will be to the lender.

Note: The APRs in the above table will be the same with any 20 year mortgage with a disclosed APR of 5% and a prepayment penalty of 4% of the amount financed – regardless of the amount financed.

Credit Card Transactions

Important: Any calculations made concerning credit cards can only be ESTIMATIONS, as credit card issuers have widely varying methods of computing monthly interest. In addition, most cards have rates that float with the prevailing cost of money and/or perceived credit risk of the borrower.

A common question you may have is how long would it take me to pay off this account, assuming I didn't make any more charges?

Let's assume your balance on the card is $750, the minimum payment is $15, and the annual rate is 12.5%.

How long would it take you to pay off the account if you paid the minimum payment each month and didn't make any more charges, and the rate didn't change?

If you considered the present value of the loan to be $750, Rate of 12.5%, with 12 end of month payments per year, compounded monthly – It would take approximately 71 months, **or almost 6 years**, with an approximate interest over the period of $314.93, or **42% of the current balance!**

"Well", you say, "I don't want to take that long to pay it off!" "Suppose I can pay $30 per month?" Dividing $30 into $750 gives 25 months, but doesn't account for the interest. In order to pay it off in 25 months, you would have to pay $30, **plus the monthly interest** each month.

This calculation would involve Equal Principal Payments, described previously. You will find that the payments will actually range from a high of approximately $37.81, on the first payment, down to $30.31 on the final payment. The total interest paid over the period would be approximately $101.56.

Lotteries

Lotteries provide an interesting example for time value calculations. Most lotteries give a winner the choice of taking the prize in annual instalments, or a lump sum immediately.

A recent multi-state lottery with a jackpot of $12 million offered the winner the choice of taking 30 annual payments of $400,000, or a lump sum of $6,700,000.

In order to insure the ability to pay the winner if he chooses the annual payment option the lottery commission typically purchases an annuity contract from an insurance company. The cost of the annuity is the amount the commission pays to the winner if he chooses the lump sum option.

Therefore, if the winner chooses the annual payment option he is, in effect, lending the insurance company $6,700,000 in return for the payment by the insurance company of the 30 annual instalments of $400,000. The difference between the PRESENT VALUE ($6,700,000) and the sum of the instalments ($12 Million) is the interest earned.

Using standard interest calculations for a series of 30 annual instalments of $400,000 with the instalments at the beginning of the payment periods (the winner receives the first instalment immediately) and assuming that the winner would want his interest compounded semi-annually, the interest rate he is earning is 4.597%.

So, one factor the winner would have to decide is whether he thinks he could get a better return if he invested the $6,700,000 elsewhere.

> Of course, this would be only one factor the winner should consider. Probably more important would be the tax considerations of the two options as well as his age in relation to his estate planning.

Comments

In the early 80s I started developing a computer program to perform various calculations dealing with the time value of money. This was done as an exercise to help me learn programming as well as the various formulas and methods associated with those calculations.

The program was originally written in the original BASIC programming language. Later, Microsoft came out with an improved version of BASIC called QUICK BASIC and I rewrote the program in that language. Then, around 1995, when Microsoft came out with the Windows operating system I rewrote the program again using Microsoft's VISUAL BASIC, which took advantage of the Windows format. Along the way I kept adding to the program.

Since I started this exercise many programs have come out dealing with instalment loans, leases, etc. and most spreadsheet programs contain many standard financial functions. However, none of the applications I have seen allow for variations common in actual practice, especially the concept of variable payment periods and compounding periods that may not occur at the same time as the payment periods.

This document was adapted from the Help files associated with that program. I wrote the program for my amusement and education and have no intention of distributing it; however I thought this summary of the elements of Time Value of Money might be interesting to someone who may be thinking of borrowing or lending money.

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